Math 53, Discussions 116 and 118 **2022-09-12**

More polar and vectors

Answers included

Questions

Question 1. Find the angle between the vector $-\mathbf{i} + \sqrt{3}\mathbf{j}$ and the positive x-axis.

Question 2. Let $\mathbf{v} \in \mathbb{R}^2$ be the vector $\langle 3, 7\rangle$. How many unit vectors $\mathbf{u} \in \mathbb{R}^2$ form an angle of $\pi/3$ with the vector $\mathbf{v}$?

Let $\mathbf{v} \in \mathbb{R}^3$ be the vector $\langle 2, 4, 5\rangle$. How many unit vectors $\mathbf{u} \in \mathbb{R}^3$ form an angle of $\pi/3$ with the vector $\mathbf{v}$?

Note: the specific coordinates of the vector $\mathbf{v}$ are not actually relevant to the problem; any other nonzero vector would work just as well. Also you are not asked to compute $\mathbf{u}$, only to say how many solutions there are.

Question 3. The polar curve $r = 2 + \cos(3\theta/2)$ has three points of self-intersection; see Figure 1. Find the (x, y)-coordinates of the one in the first quadrant, as well as the slopes of the two tangent lines at that point.

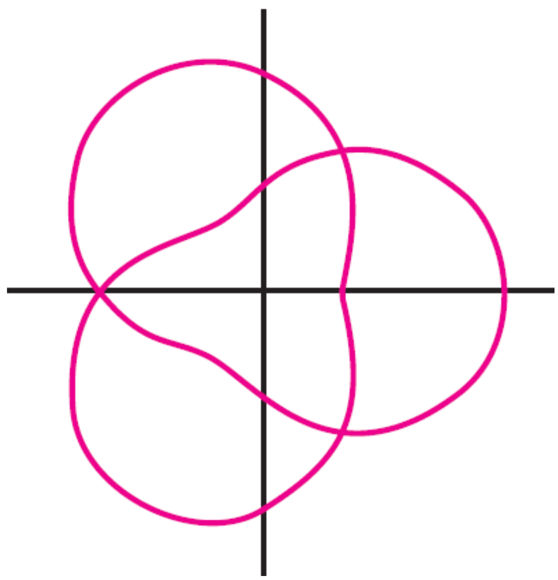

Figure 1. Copied from Stewart.

Below are brief answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

Answers to questions

Question 1. $2\pi/3$.

Question 2. Two possibilities in $\mathbb{R}^2$, and infinitely many possibilities in $\mathbb{R}^3$.

Question 3. $(-2, 0)$, $(1, \sqrt{3})$, $(1, -\sqrt{3})$ are the three intersection points; the one asked for is $(1, \sqrt{3})$. (Deduce the θ values either from rotational symmetry or by noting that, if the curve reaches an intersection point at some θ, then it reaches it again at $\theta + 2\pi$. Then solve $2 + \cos(3\theta/2) = 2 + \cos(3(\theta + 2\pi)/2)$.)

The tangents are:

$$y - \sqrt{3} = \left(\frac{16}{13} - \frac{25}{39}\sqrt{3}\right)(x - 1)$$
$$y - \sqrt{3} = \left(-\frac{16}{13} - \frac{25}{39}\sqrt{3}\right)(x - 1)$$

(Use the slope formula; first plug in $\theta = \pi/3$ and then $\theta = 7\pi/3$ because those correspond to the different "branches" at the intersection point.)